

DEUS
DEUS
DEUS
DEUS

Non-linear Matter Power Spectrum Covariance and Cosmological Parameter Errors

Pier-Stefano Corasaniti

LUTH, CNRS & Obs. Paris

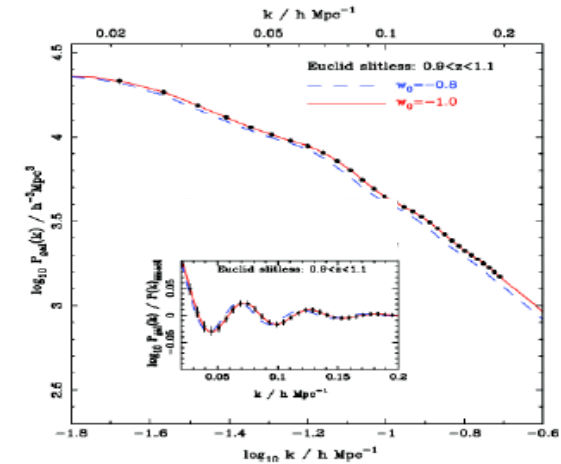
L. Blot, PSC, L. Amendola, T. Kitching, arXiv:1512.05383

L. Blot, PSC, J.-M. Alimi, V. Reverdy, Y. Rasera, arXiv:1406.2713

Galaxy Clustering Measurements

Expectations

- Large Volumes & Wide Redshift Range
- High Galaxy Number Density
- Reach \sim few % Statistical Errors
- High-precision Cosmology with BAO or Galaxy Power Spectrum



from Euclid red book

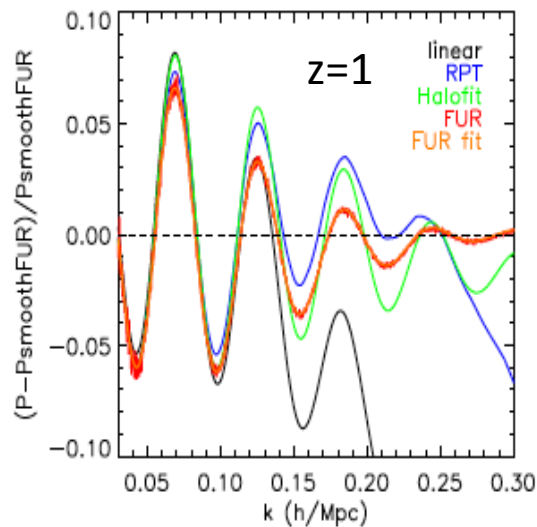
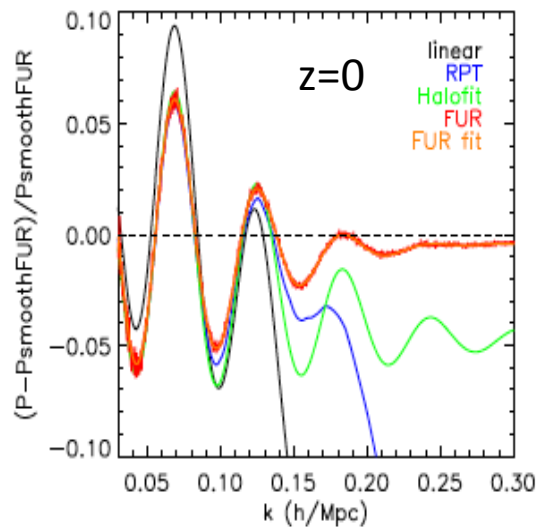
Theoretical Challenges

- Cosmological model predictions to few % accuracy
- Estimate Power Spectrum Covariance

Why Challenging?

Non-Linear Dynamics

- At % level non-negligible even at $k \sim 0.15$ and $0 < z < 1$



BAO spectrum
from DEUS-FUR
LCDM-WMAP5
simulation

Rasera et al. (2014)

- Non-linear mode correlations
- Deviations from Gaussian statistics

Power Spectrum Covariance

Non-Linear Contribution

$$\text{cov}(k_1, k_2) = \frac{2}{N_{k_1}} P^2(k_1) \delta_{k_1 k_2} + \frac{1}{V} \int_{\Delta_{k_1}} \int_{\Delta_{k_2}} \frac{d^3 k_1'}{V_{k_1}} \frac{d^3 k_2'}{V_{k_2}} T(k_1', -k_1', k_2', -k_2')$$

- Non-linear regime sources non-Gaussianity ($T \neq 0$)
- Fully analytical trispectrum is not viable (several models on the market still require simulation input)

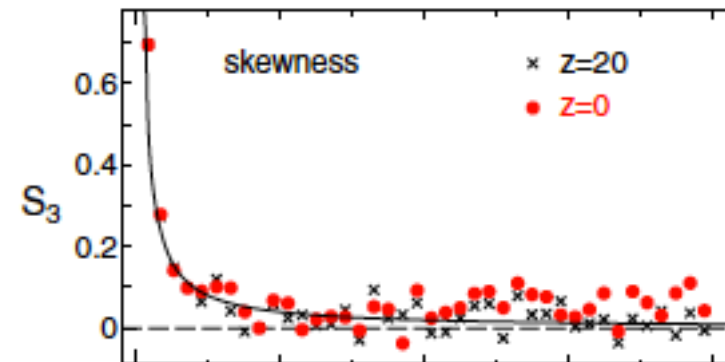
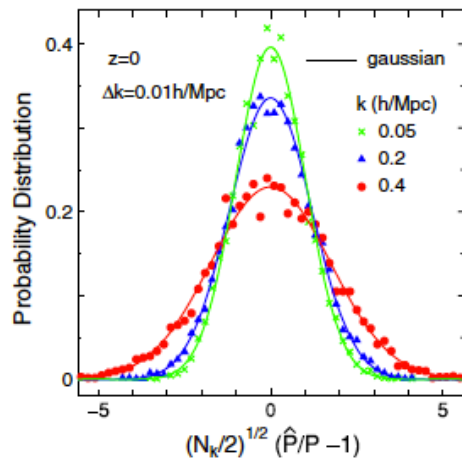
Sampling N-body Ensemble

$$\text{cov}(k_1, k_2) = \frac{1}{N_r - 1} \sum_{i=1}^{N_r} \left[\hat{P}_i(k_1) - \bar{P}(k_1) \right] \left[\hat{P}_i(k_2) - \bar{P}(k_2) \right] \quad \bar{P}(k) = \frac{1}{N_r} \sum_{i=1}^{N_r} \hat{P}_i(k)$$

Previous Studies

Power Spectrum Statistics

- $N_r = 5000$ N-body PM simulations
- $L_{\text{box}} = 1 \text{ Gpc } h^{-1}$ & $N_p = 256^3$ ($m_p = 4.1 \times 10^{12} h^{-1} M_\odot$)



Takahashi et al. (2014)

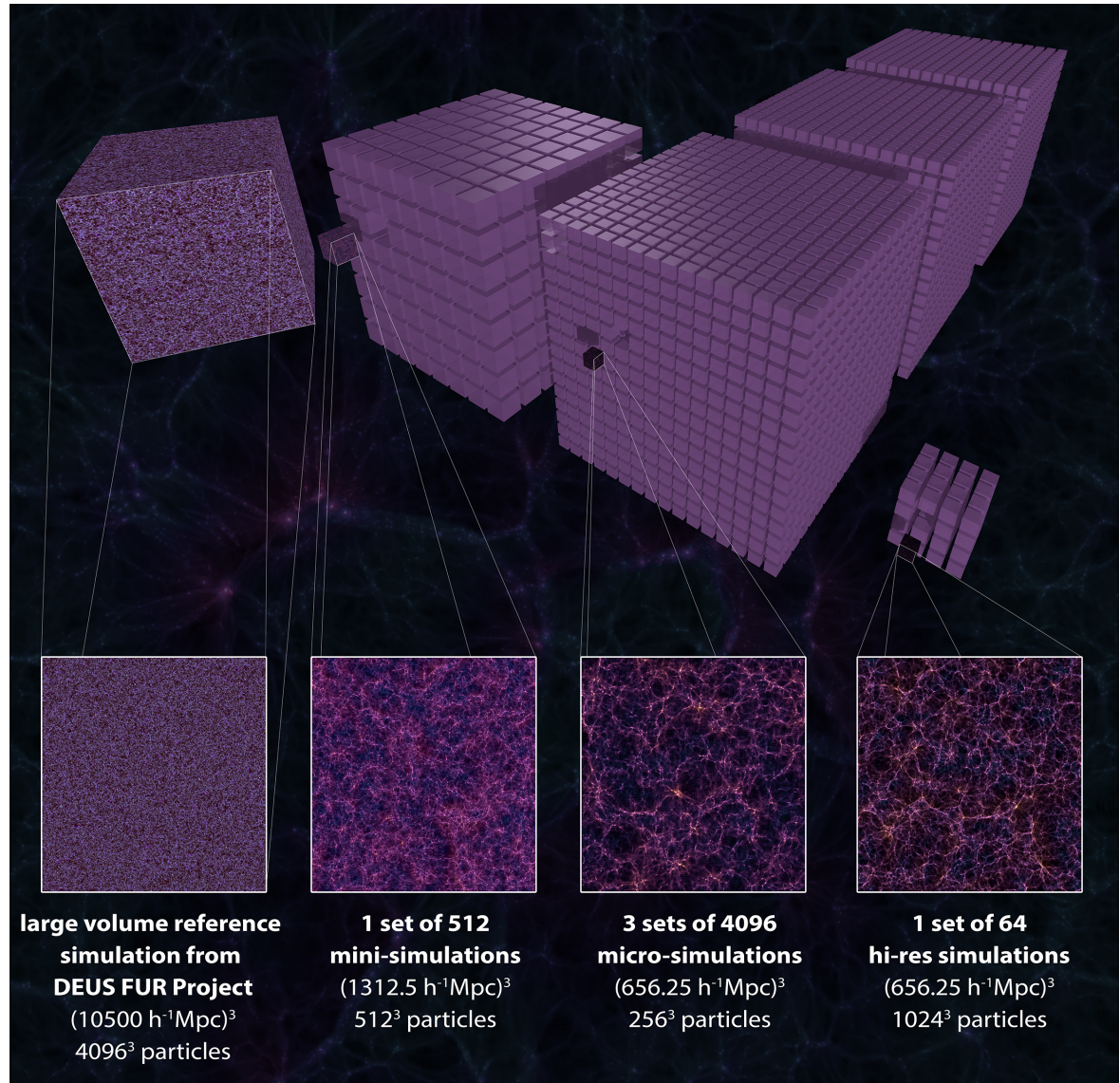
- Statistically consistent with Gaussian expectations
- Not conclusive (still large statistical uncertainties)

DEUS-Parallel Universe Runs

$N_r = 12288$ Simulations
 $N_p = 256^3$ $L_{\text{box}} = 648$ Mpc/h
 $m_p = 1.2 \times 10^{12} M_{\text{sun}}$

$N_r = 512$ Simulations
 $N_p = 512^3$ $L_{\text{box}} = 1.3$ Gpc/h
 $m_p = 1.2 \times 10^{12} M_{\text{sun}}$

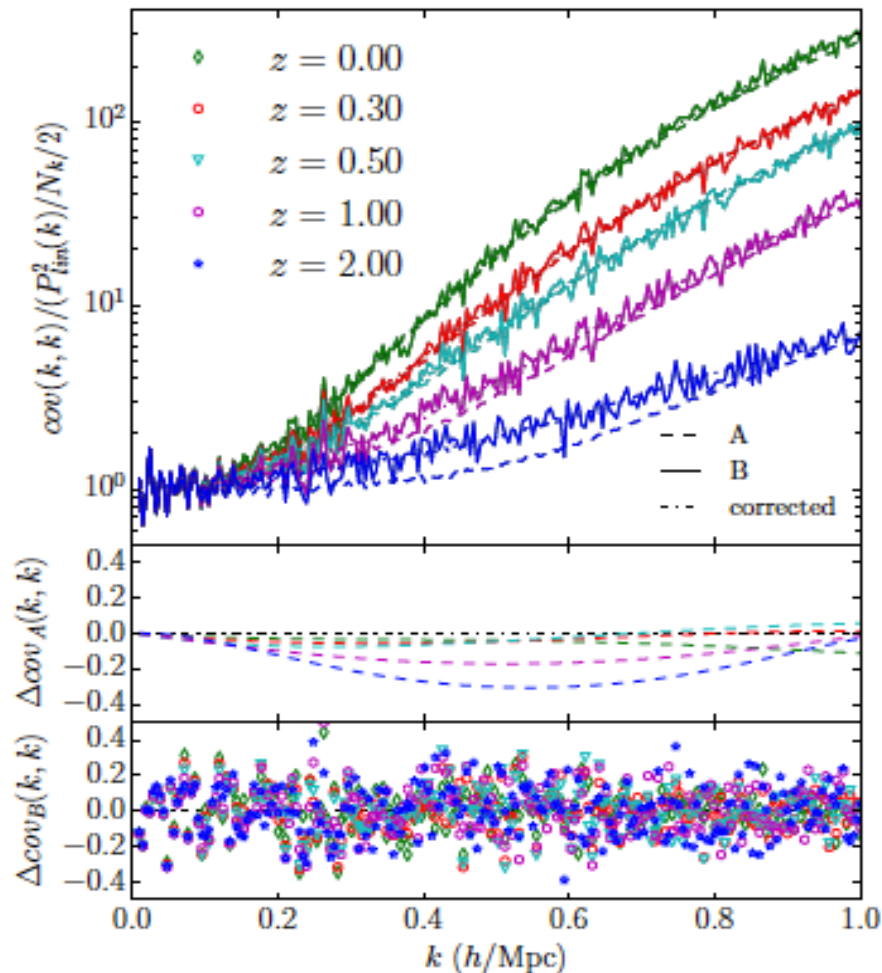
$N_r = 96$ Simulations
 $N_p = 1024^3$ $L_{\text{box}} = 648$ Mpc/h
 $m_p = 1.8 \times 10^{10} M_{\text{sun}}$



DEUS-PUR Covariance

Blot, PSC, Alimi, Reverdy, Rasera, arXiv:1406.2713

Mass Resolution Errors



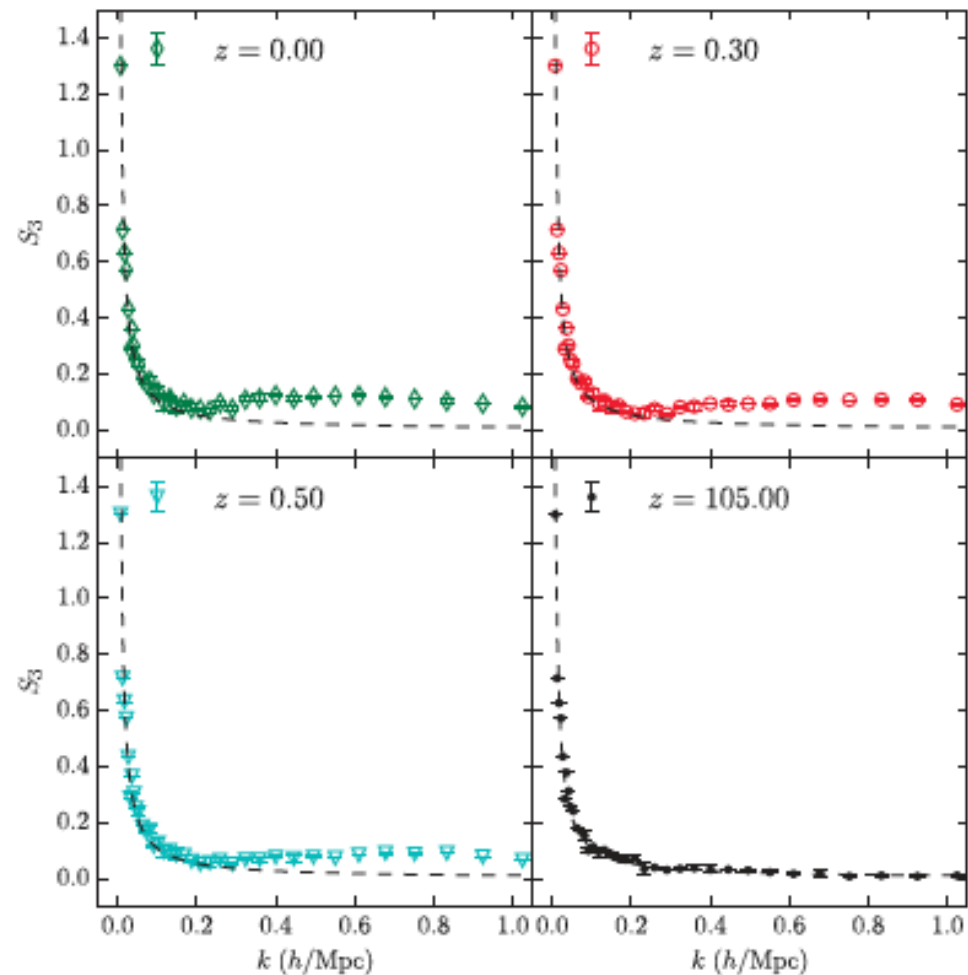
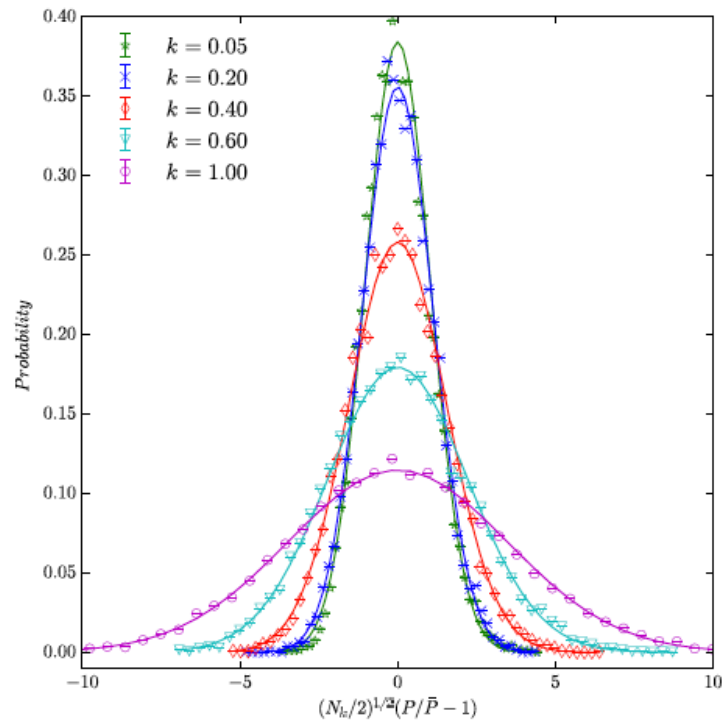
- Set A vs Set B
- At intermediate scales lower resolution leads to lower covariance
- Discrepancy decrease with z and within statistical noise for $z < 0.5$
- PM effect on trispectrum, alleviated by refinement
- Corrections

$$\hat{P}_A^{\text{corr}}(k) = \left[\hat{P}_A(k) - \bar{P}_A(k) \right] \frac{\sigma_{\hat{P}_B}(k)}{\sigma_{\hat{P}_A}(k)} + \bar{P}_B(k)$$

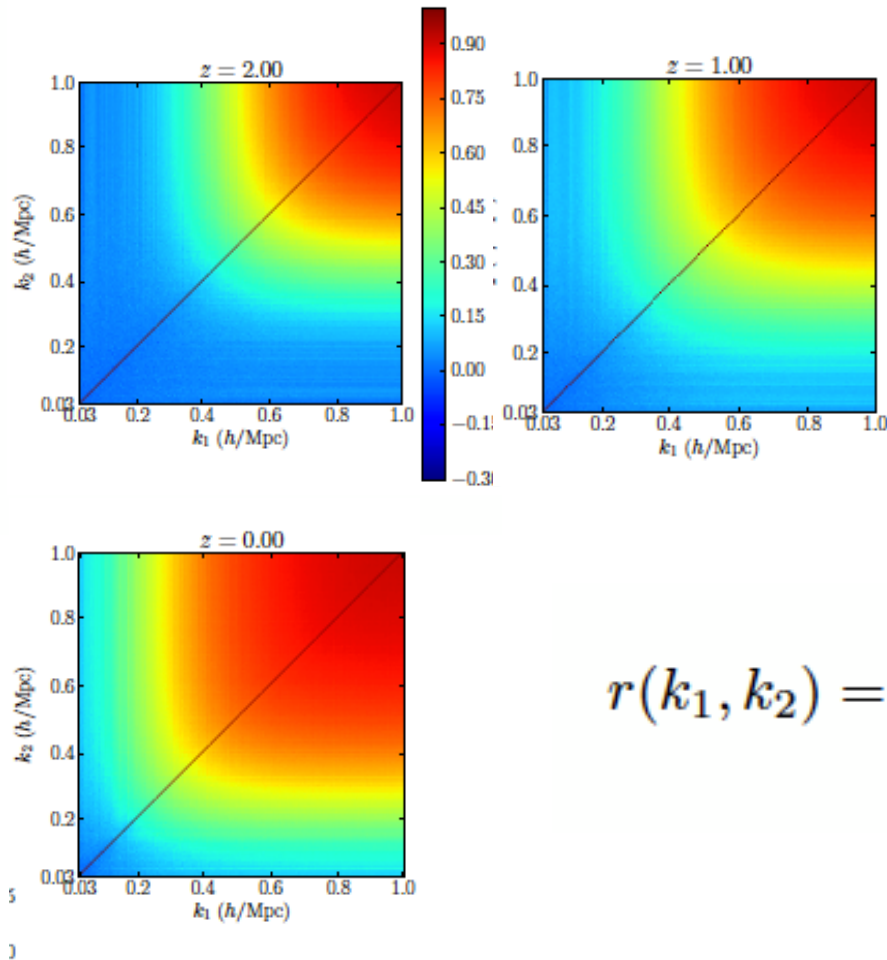
Power Spectrum Distribution

Deviations from χ^2 statistics

- $P(k)$ of Gaussian density field is χ^2 -distributed



Correlation Matrix



- Non-negligible mode correlations on BAO scales

$$r(k_1, k_2) = \frac{\text{cov}(k_1, k_2)}{\sqrt{\text{cov}(k_1, k_1) \text{cov}(k_2, k_2)}}$$

Errors on Covariance Estimation

Sampling Errors

- What is the impact of non-linearities on covariance errors?
- How errors vary as function of #-independent realizations?

Wishart Distribution

- Gaussian density field covariance:

$$p(\hat{\mathcal{C}}|\mathcal{C}, \nu, \mu) = \left(\frac{\nu^\nu \mu^{\nu/2} |\mathcal{C}|^{-\nu/2} |\hat{\mathcal{C}}|^{(\nu-\mu-1)/2}}{2^{\nu\mu/2} \Gamma_\mu[\nu/2]} \right) \exp^{-\frac{\nu}{2} \text{Tr} \hat{\mathcal{C}} \mathcal{C}^{-1}}$$

- Error scaling: $\sigma^2(\hat{\mathcal{C}}_{ij}) = \frac{1}{N_s - 1} (\langle \mathcal{C}_{ij} \rangle^2 + \langle \mathcal{C}_{ii} \rangle \langle \mathcal{C}_{jj} \rangle)$

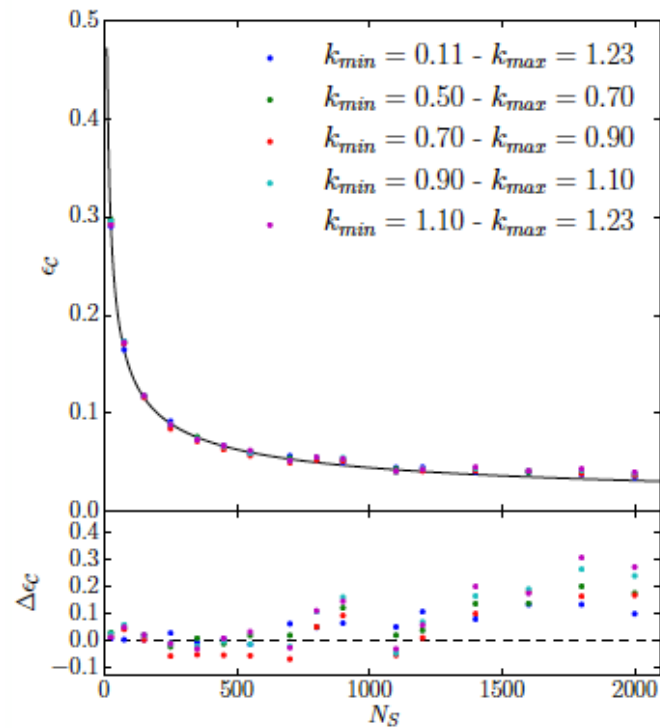
$$\epsilon_{\mathcal{C}} = \sqrt{\frac{\sum_i \sigma^2(\hat{\mathcal{C}}_{ii})}{\sum_i \langle \mathcal{C}_{ii} \rangle^2}} = \sqrt{\frac{2}{N_s - 1}}$$

Taylor, Joachimi &
Kitching (2013)

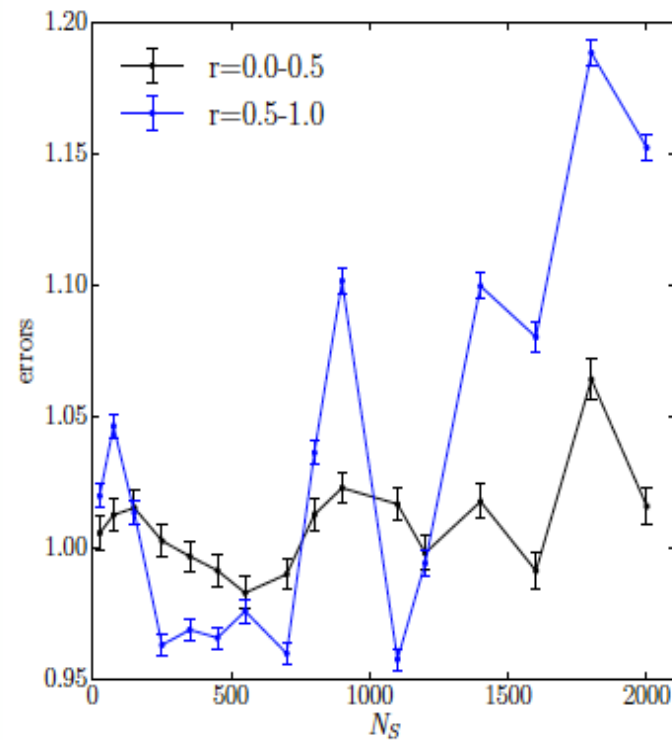
Testing Covariance Estimation Errors

Blot, PSC, Amendola, Kitching, arXiv:1512.05383

Diagonal Components:



Off-diagonal Components:



- Deviations from Gaussian predicted errors manifest only at large N_s
- Off-diag elements deviations for modes with $r > 0.5$

Fisher Forecast – Euclid-like Survey

Cosmological Parameters

$$\theta = \{\Omega_m, w, \sigma_8, n_s, \Omega_b, b_1, \dots, b_{N_z}\}$$

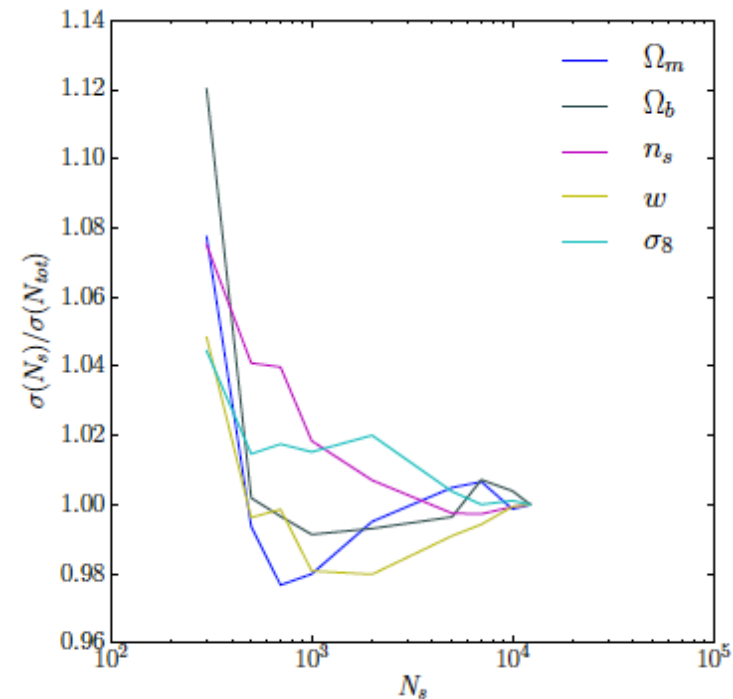
$$P_z^g = b_z^2 P_z + \frac{1}{\bar{n}_g(z)} \quad \text{cov}_z^g(k_i, k_j) = b_z^4 \hat{C}_{ij} + 2b_z^2 [P_z(k_i)P_z(k_j)]^{1/2} \bar{n}_g^{-1}(z) + \bar{n}_g^{-2}(z)$$

- 5 bins $0.5 < z < 2$
- Marginalized over constant bias b_i

z	$\bar{n}_g(z)$
0.5	4.2×10^{-3}
0.7	2.99×10^{-3}
1.0	1.81×10^{-3}
1.5	0.77×10^{-3}
2.0	0.15×10^{-3}

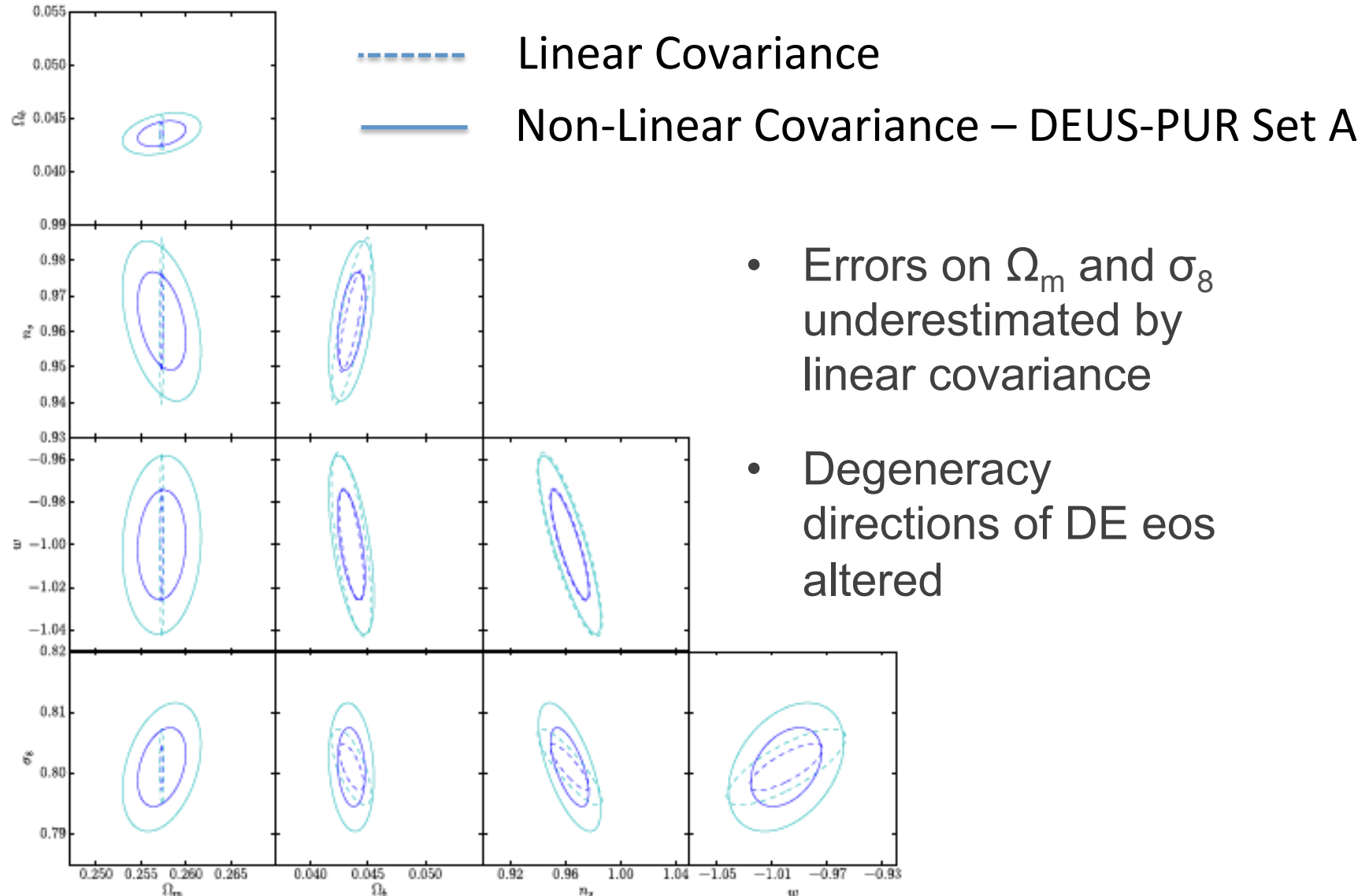
$$F_{\alpha\beta} = \sum_{l=1}^{N_z} \sum_{i,j=1}^{N_d} \frac{\partial P_{z_l}^g}{\partial \theta_\alpha}(k_i) \frac{\partial P_{z_l}^g}{\partial \theta_\beta}(k_j) \text{cov}_{z_l}^g(k_i, k_j)$$

- Errors convergence for large N_r
- For $N_r > 5000$ Covariance Estimation Errors $< 1\%$



Blot, PSC, Amendola, Kitching, arXiv:1512.05383

The Impact of Non-Linearities

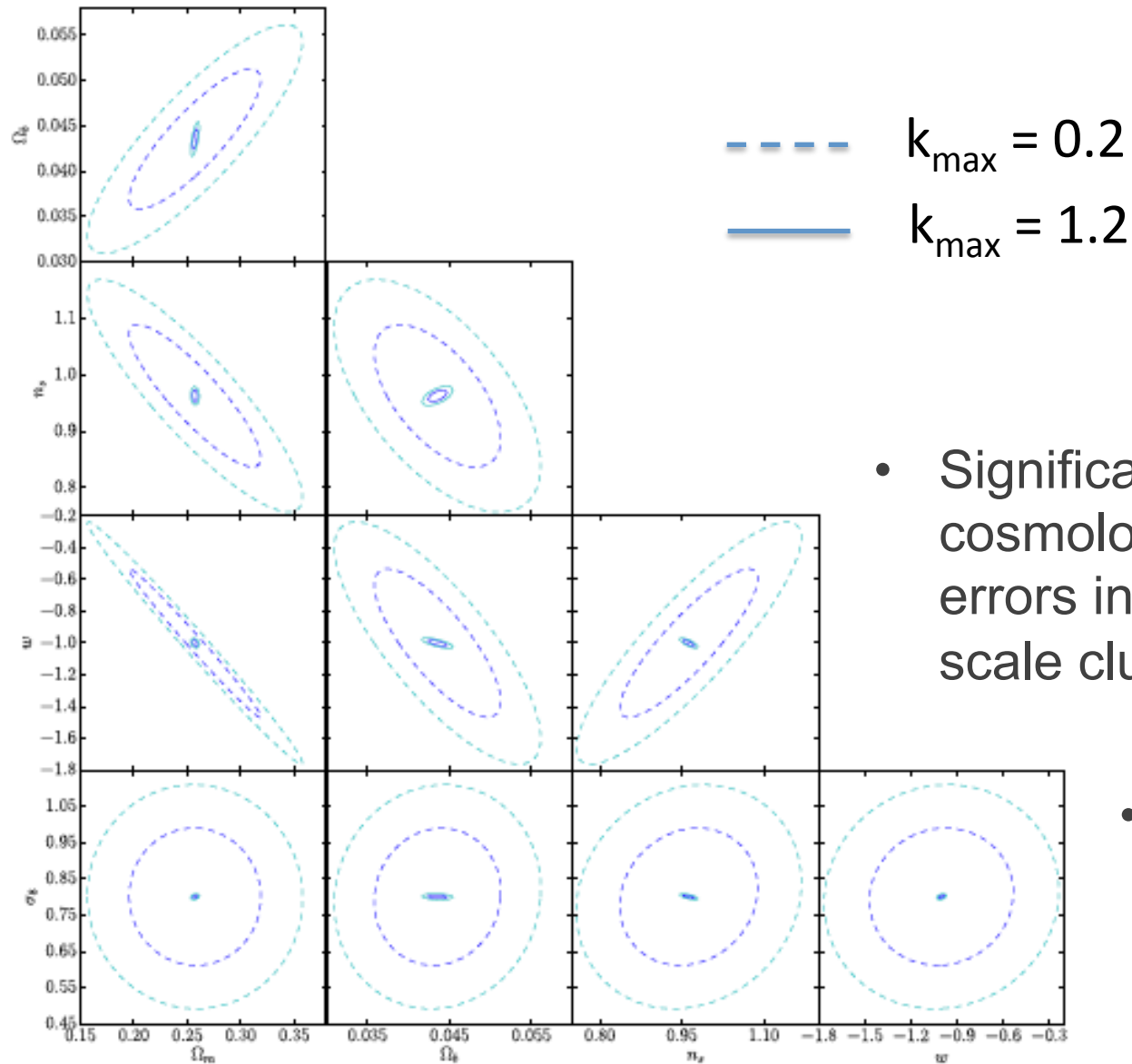


- Errors on Ω_m and σ_8 underestimated by linear covariance
- Degeneracy directions of DE eos altered

Conclusions

- Accurate Covariance Estimation is Key to Perform unbiased Power Spectra Data Analysis
- Non-Linearities of the matter density field induce non-Gaussian Errors requiring large simulation ensemble (>5000) to accurately sample covariance and reduce estimation errors $<1\%$
- Gaussian covariance significantly underestimate cosmological parameter uncertainties
- Covariance Matrix & Python Libraries available at <http://luth.obspm.fr/~luthier/blot>

Probing non-linear scales



- Significant improvement of cosmological parameter errors in probing small scale clustering

- Require more investigation to account baryonic effects