2D Mass Mapping

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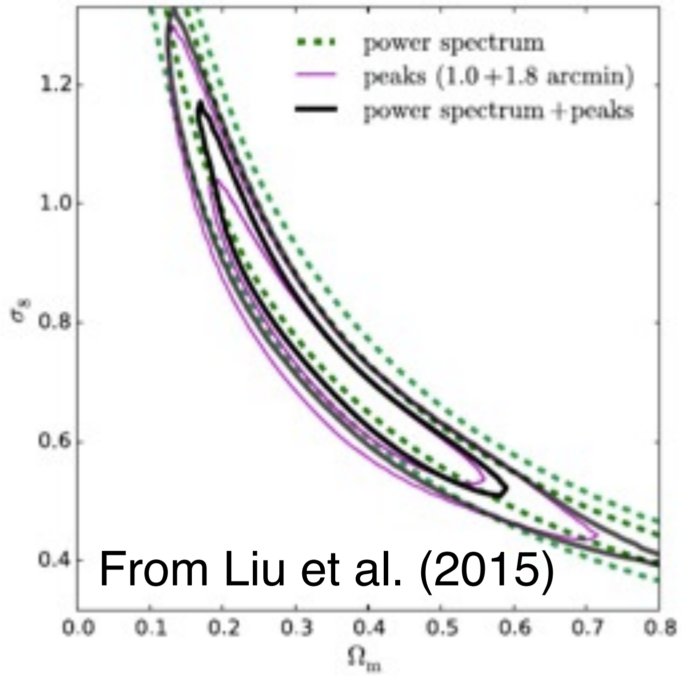
Convergence Map

- **Mass Mapping**: **2D-MASS-WL**, 3D-MASS-WL
  - Originally, mass maps were considered not scientifically useful, but the situation is now clearly different.
  - The field is evolving, and several 2D and 3D codes now exist.
  - Science case work ongoing, and most requirements are not defined



Cosmo. Parameters

Clusters

$$P_1(k) = \frac{k_1^2 - k_2^2}{k^2}$$

$$P_2(k) = \frac{2k_1 k_2}{k^2}$$

$$\hat{\kappa} = P_1 \hat{\gamma}_1 + P_2 \hat{\gamma}_2$$

=> PROBLEMS: Noise + Irregular Sampling

Mass mapping as an inverse problem

Binned data: $\gamma = F^* P F \kappa$

Unbinned data: $\gamma = T^* P F \kappa$

T = Non Equispaced Discrete Fourier Transform (NDFT)

$$\min_{\kappa} \frac{1}{2} \parallel \gamma - \mathbf{P}\kappa \parallel_2^2 \quad \text{with} \quad \mathbf{P} = T^* P F$$

$$g = \frac{\gamma}{1-\kappa} \longrightarrow \min_{\kappa} \frac{1}{2} \parallel (1-\kappa) g - \mathbf{P}\kappa \parallel_2^2$$

$L = \mathbf{T}^*\mathbf{PF}$ is not directly invertible $\Rightarrow$ **Linear inverse problem.**

2D Mass Mapping Algorithms

Linear Methods:

- Kaiser-Squires (1993) + Gaussian smoothing

Non Linear methods:

- **For clusters:**
  - Model fitting algorithms (Bartelmann et al, 1996; Bradac et al, 2005; Jullo and Kneib, 2009).
  - Aperture Mass (Seitz and Schneider, 1996; 2002)
- **For larger fields:**
  - Maximum Likehood (Bartelmann et al, 1996)
  - MemLens (Bridle et al, 1998; Marshal and Hobson, 2002)
  - FastLens + MR-Lens (Starck, Pires, Refregier, 2006; Pires et al, 2009)
  - **Glimpse2D** (Lanusse, Starck, Leonard, Pires, in preparation).

Aperture Mass and Wavelets

$$M_{ap}(\boldsymbol{\theta}) = \int d^2\boldsymbol{\vartheta}\ \gamma_t(\boldsymbol{\vartheta}) Q(|\boldsymbol{\vartheta}|)$$

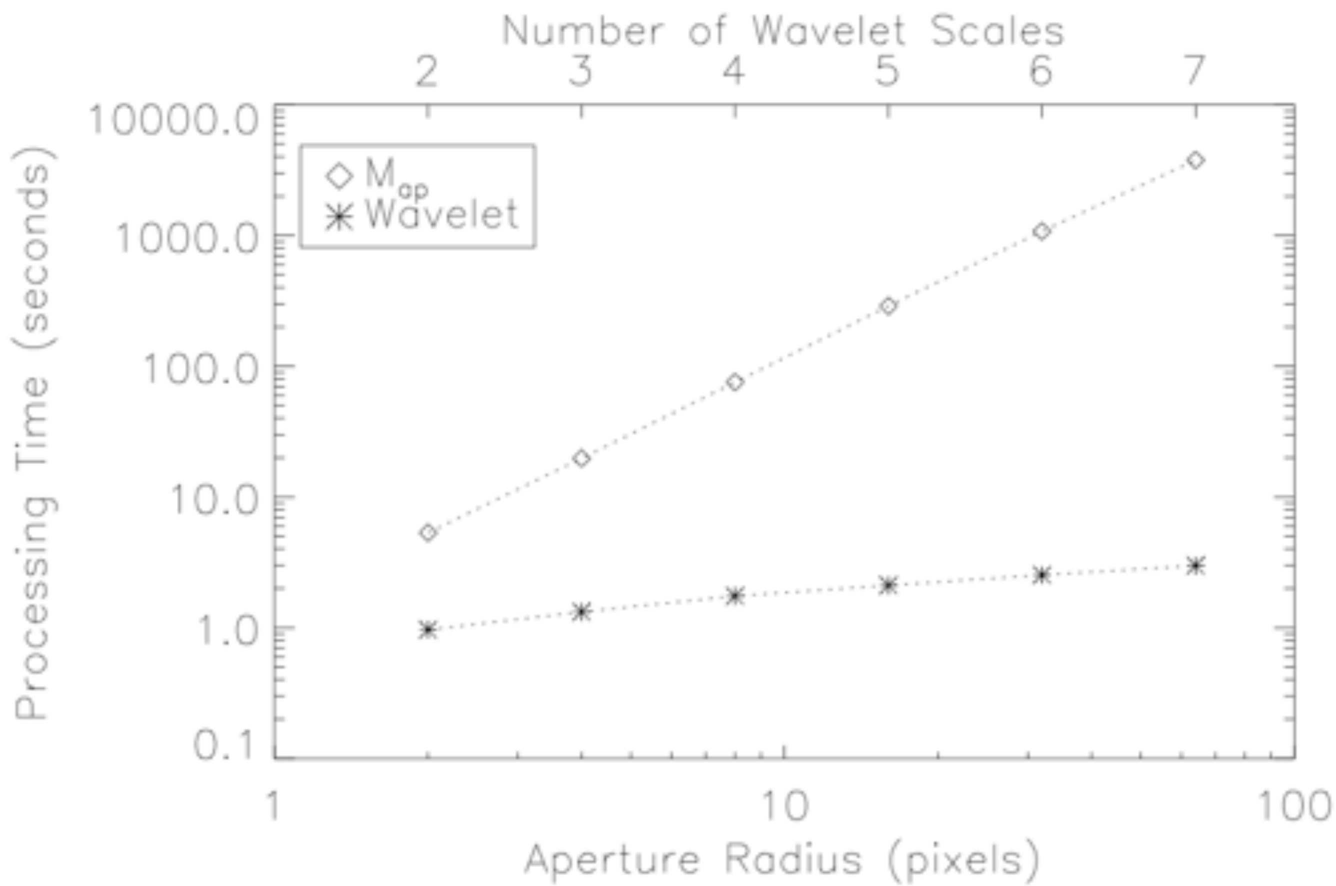
$$M_{ap}(\theta) = (\mathbf{\Phi}^t \kappa)_\theta$$

$\Rightarrow$ Wavelets filters are formally **indentical** to Mass aperture

A. Leonard, S. Pires, J.-L. Starck, "Fast Calculation of the Weak Lensing Aperture Mass Statistic", **MNRAS**, 423, pp 3405-3412, 2012.

but wavelets presents many advantages:
- compensated and **compact** support filters
- **fast** calculation:
- **all scales** processed in one step.
- **reconstruction** is possible

  ==> image restoration for peak counting



Sparsity and Mass mapping

Mass-Shear: $\gamma = \mathbf{P}\kappa$ with $\mathbf{P} = T^* P F$

T = Non Equispaced Discrete Fourier Transform (NDFT)

$$\min_{\kappa} \frac{1}{2} \parallel \gamma - \mathbf{P}\kappa \parallel_2^2$$

sparse regularizaton

$$\min_{\kappa} \frac{1}{2} \parallel \gamma - \mathbf{P}\kappa \parallel_2^2 + \lambda \parallel \mathbf{\Phi}^t \kappa \parallel_1$$

$$g = \frac{\gamma}{1-\kappa} \longrightarrow \min_{\kappa} \frac{1}{2} \parallel (1-\kappa) g - \mathbf{P}\kappa \parallel_2^2 + \lambda \parallel \mathbf{\Phi}^t \kappa \parallel_1$$

=> Write the mass-mapping as a single optimization problem with a ***multi-scale sparsity prior*** addressing all these issues (i.e. reduced shear, missing data, noise).

The 2D Glimpse Algorithm

$$\min_{\kappa} \frac{1}{2} F(\kappa) + \lambda \parallel \mathbf{\Phi}^t \kappa \parallel_1 \quad \text{with} \quad F(\kappa) = \frac{1}{2} \parallel (1-\kappa) g - \mathbf{P}\kappa \parallel_2^2$$

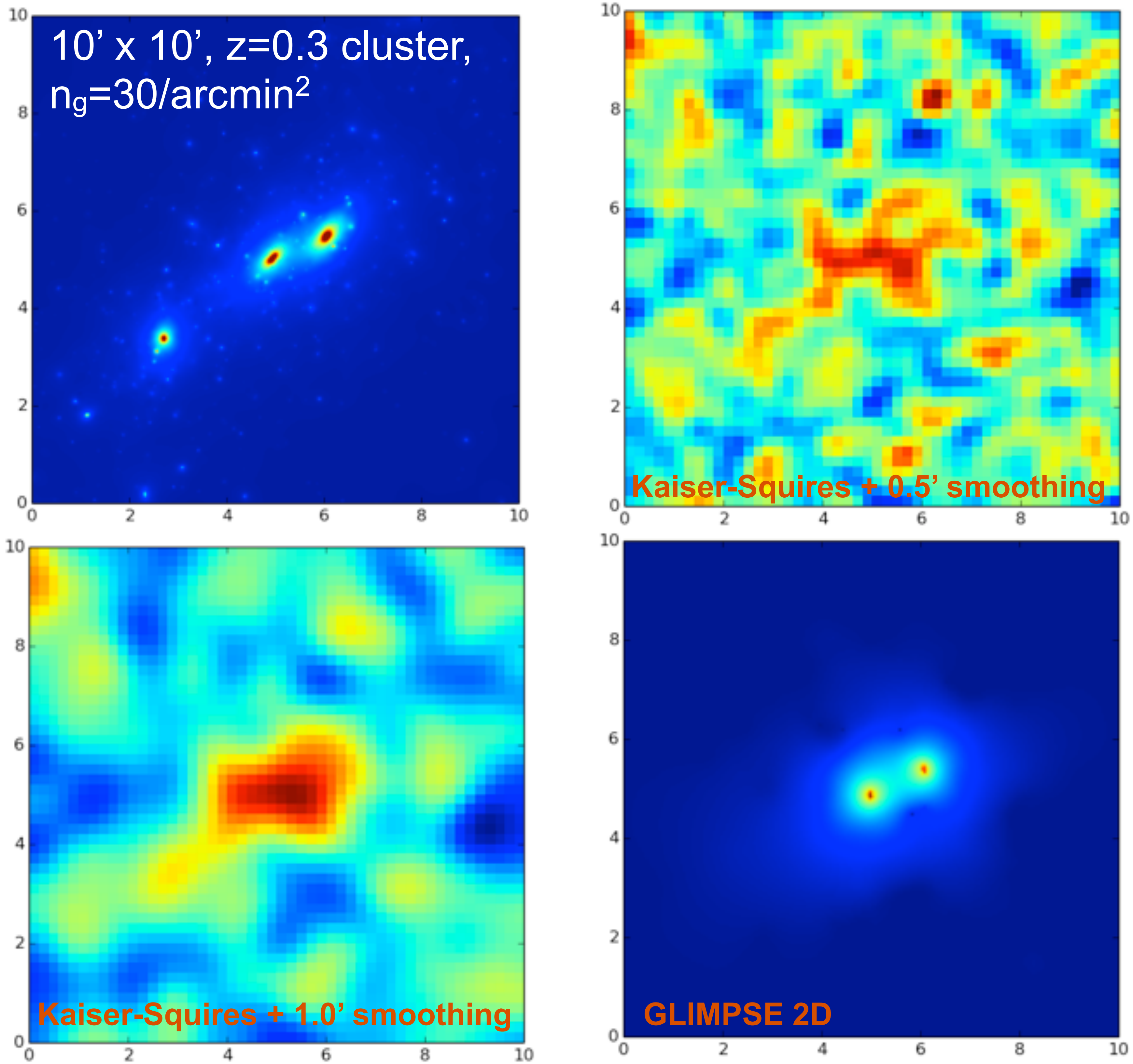
Primal-dual splitting:

$$\begin{cases} \kappa^{(n+1)} & = & \kappa^{(n)} + \tau \left(\nabla F(\kappa^{(n)}) + \mathbf{\Phi} \alpha^{(n)} \right) \\ \alpha^{(n+1)} & = & (\mathrm{Id} - \mathrm{ST}_\lambda) \left(\alpha^{(n+1)} + \mathbf{\Phi}^t (2\kappa^{(n+1)} - \kappa^{(n)}) \right) \end{cases}$$

adapted from Vu (2013)

- Fast and flexible algorithm
- Sparsity constraint λ estimated locally by noise simulations $\Longrightarrow$ Accounts for **survey geometry**, **varying noise levels**

Missing Data + Noise



Flexion

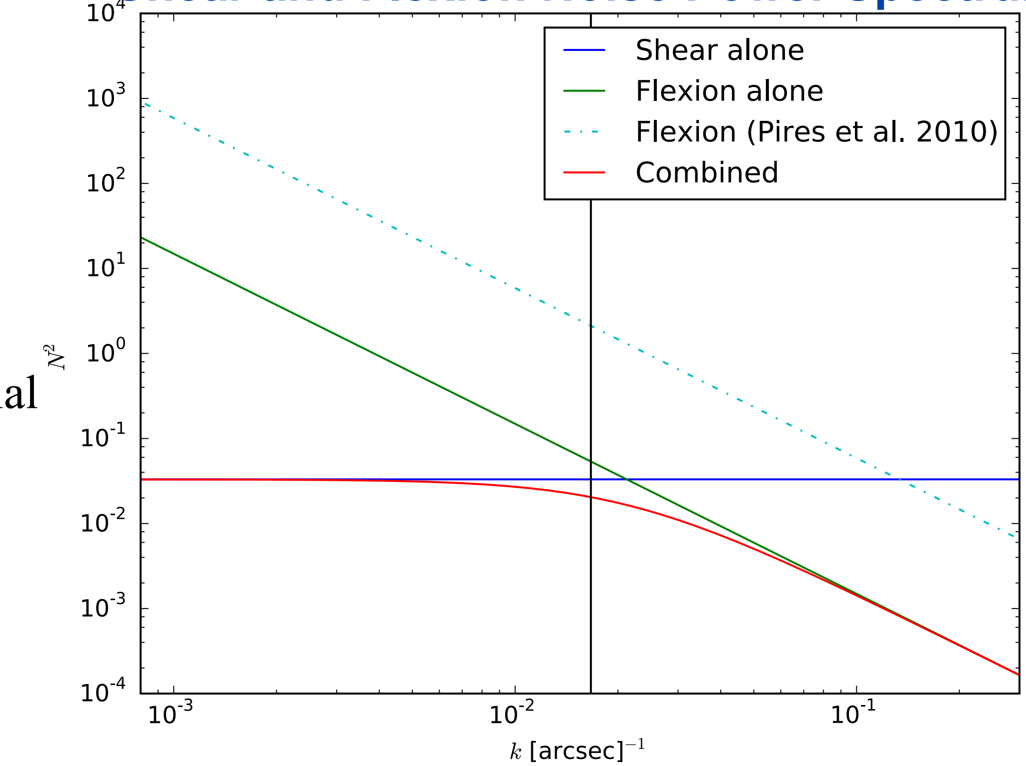
Shear is noise dominated on small scales ==> Substructures are lost

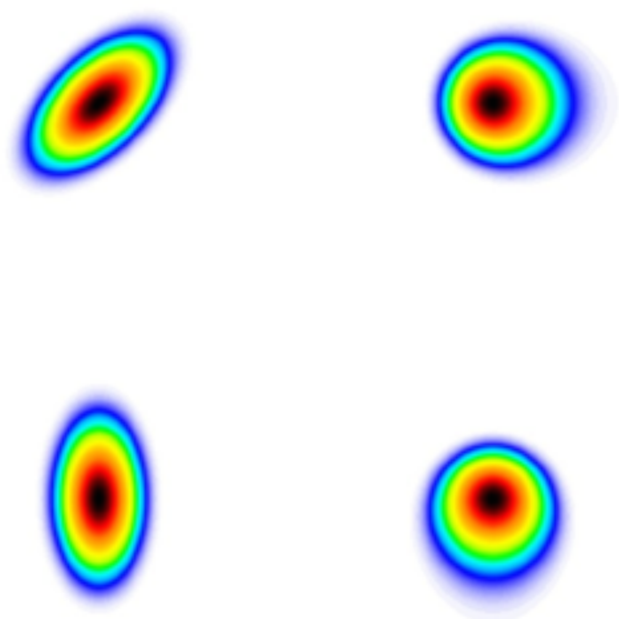
Small-scale substructure can be recovered from strong lensing when available.

Gravitational **Flexion** is useful in the intermediate regime.

Flexion gives information relative to the third order derivatives of the lensing potential

Shear and Flexion Noise Power Spectrum





Shear (left) and first flexion (right) (Bartelmann 2010)

$$\mathcal{F} = \nabla \kappa$$

$$F = \frac{\mathcal{F}}{1-\kappa}$$

Flexion Information

We can integrate flexion in our reconstruction framework

=> **Jointly** fit shear and flexion

$$\min_{\kappa} \frac{1}{2} \parallel (1-\kappa) \begin{bmatrix} g \\ F \end{bmatrix} - \begin{bmatrix} \mathbf{P} \\ \mathbf{Q} \end{bmatrix} \kappa \parallel_2^2 + \lambda \parallel \mathbf{\Phi}^t \kappa \parallel_1$$

=> **Jointly** fit shear and flexion with redshift information

$$\min_{\kappa} \frac{1}{2} \parallel (1-\mathbf{Z}\kappa) \begin{bmatrix} g \\ F \end{bmatrix} - \mathbf{Z} \begin{bmatrix} \mathbf{P} \\ \mathbf{Q} \end{bmatrix} \kappa \parallel_2^2 + \lambda \parallel \mathbf{\Phi}^t \kappa \parallel_1$$

with $\mathbf{Z} = \Sigma_{critic}^{\infty} / \Sigma_{critic}(z_i)$

$$\Sigma_{crit}^{\infty} = \lim_{z \to \infty} \Sigma_{crit}(z)$$

$$\Sigma_{crit}(z_s) = \frac{c^2}{4\pi G} \frac{D_S}{D_L D_{LS}}$$

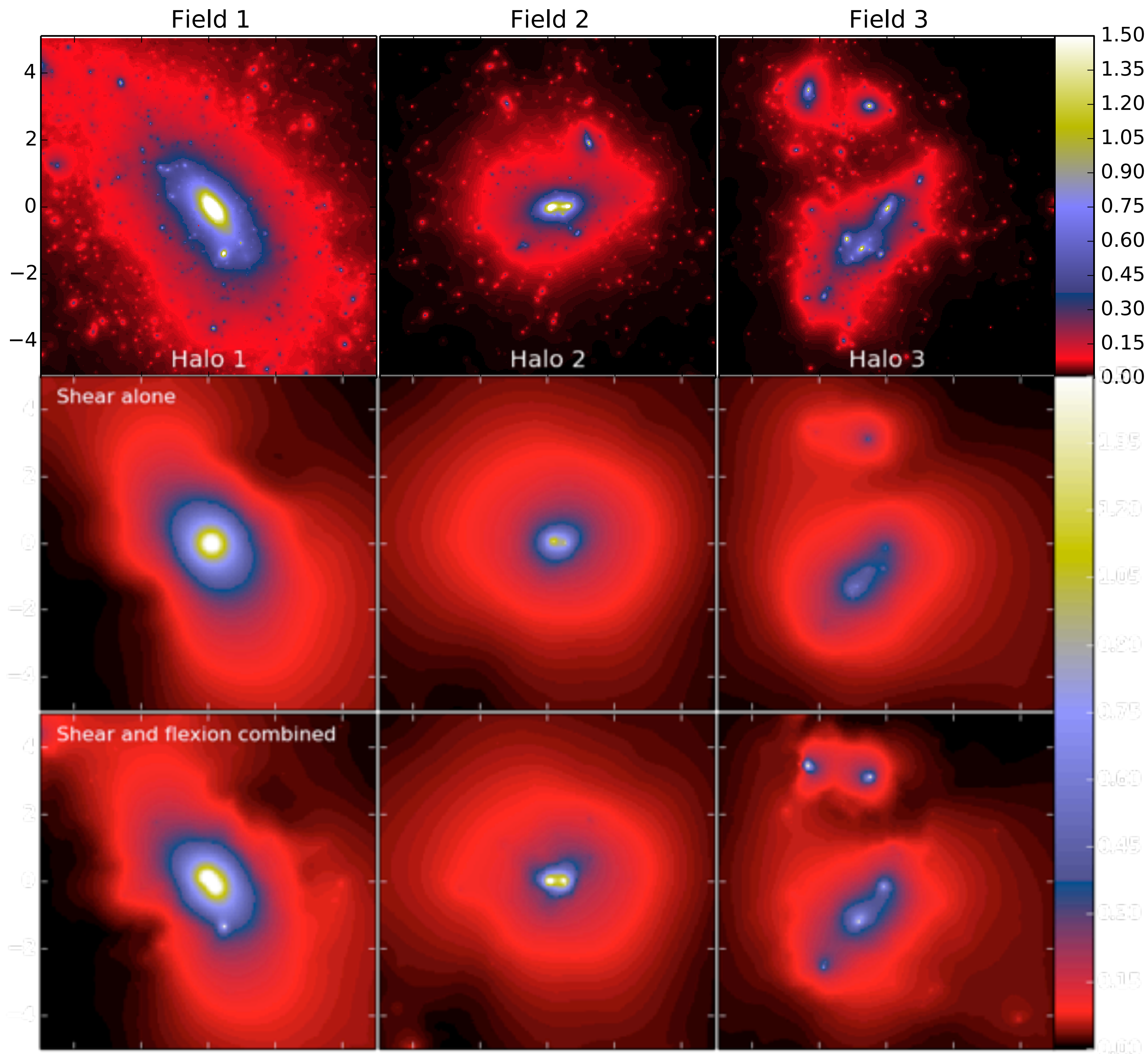
Individual redshifts have two benefits:

- Directly map the **surface mass density** of the lens
- Mitigate the **mass-sheet degeneracy** when κ becomes significant (Bradac et al. 2004)

Simulations with Flexion

Simulate reduced flexion

Flexion noise $\sigma_F = 0.029$ arcsec^{-1} (Cain et al, 2011)



Reconstruction from one realisation

Flexion

Benefits of adding flexion:

- Improvement on the recovered profiles below 0.5 arcmin
- Recovery of small-scale substructure at the 10 arcsec scale

Conclusions

* **GLIMPSE2D**: A new mass mapping algorithm, based on sparsity and proximal optimization theory:

- Does not require angular binning of the ellipticities
- Accounts for reduced shear
- Proper regularization of missing data

A new framework :

=> Can include individual redshift PDFs of sources

•Directly map the **surface mass density** of the lens

•Mitigate the **mass-sheet degeneracy** when the convergence becomes significant (Bradac et al. 2004)

=> Can include flexion measurements if available

⇒ Can be also be used for non-parametric high-resolution cluster density mapping from weak lensing alone

Lanusse F., Starck J.-L., Leonard A., and Pires S. (2015), High Resolution Weak Lensing Mass Mapping combining Shear and Flexion , in prep.

- **Bridge** between **low resolution** weak lensing and high resolution **strong** lensing
- Can recover cluster **substructures** without strong lensing information
- Ideal tool for investigating **models of dark matter**

* **The science case is not yet mature: no requirement.**